Chain rule with trig functions

Let's look at how chain rule works in combination with trigonometric functions, keeping in mind that everything we've learned about power rule, product rule, and quotient rule still applies.

Example

Use chain rule to find the derivative.

$$y = \sin 8x^2$$

Using substitution, we see that $u = 8x^2$ and $u' = 16x$.

Our original equation becomes

$$y = \sin u$$

and the derivative is

$$y' = \cos u(u')$$

Back-substituting for u and u', we get

$$y' = \cos 8x^2(16x)$$

$$y' = 16x \cos 8x^2$$

Let's look at another example.

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Example

Use chain rule to find the derivative.

$$y = \left(\tan 3x^4\right)^3$$

In this case we want to use a double substitution, where

$$u = 3x^4$$

$$u' = 12x^3$$

and

$$v = \tan u$$

$$v' = \sec^2 u(u')$$

Our original equation is

$$y = [v]^3$$

and its derivative is

$$y' = \left(3v^2\right) v'$$

Back-substituting for v and v', we get

$$y' = \left[3(\tan u)^2\right] \left[\sec^2 u(u')\right]$$

Back-substituting again, but this time for u and u', we get

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$$y' = \left[3\left(\tan 3x^4\right)^2\right]\left[\sec^2 3x^4\left(12x^3\right)\right]$$

$$y' = 36x^3 \tan^2 3x^4 \sec^2 3x^4$$

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